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ELECTROPRODUCTION AT VERY SMALL VALUES OF THE SCALING VARIABLE*

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I. Introduction

The description of electroproduction dynamics at very small values of the scaling variable x (I am thinking of $x < 10^{-2}$) poses special challenges for theory. The issues are quite complementary to those at large x, where analyses of structure functions in terms of moments provide a direct link to the small-distance structure of current correlation functions, and nearly incontrovertible links to QCD predictions. At wee x, the process involves large longitudinal distances along the light cone[1] and hence issues of the geometry of the collision process, including A-dependence and morphology of the final-state hadron phase-space distribution.

Furthermore, perturbative QCD calculations show[2] that higher order processes are asymptotically very important; there are a plethora of contributions of order $(\alpha_s \log^2)^n$. Thus the Q^2 and x dependences should exhibit strong scaling violations (on logarithmic scales).

At present, the limitation of $x \lesssim .02$ implies $Q^2 \lesssim$ 1-10 GeV^2. The future of such studies with CERN or Fermilab neutrino beams is therefore somewhat limited, although the Fermilab muon beam should be quite useful in extending our understanding of this kinematic regime. But this situation in principle changes dramatically at HERA, where the leverage in Q^2 goes out to $\lesssim$ 1000 GeV^2. The HERA events at x ~ .02 have the kinematic structure of a 30 GeV electron scattering at large angles from a parton in the 10 GeV momentum range. These appear in principle very accessible. But, for the time being, the Q^2 leverage is not so great and we will set aside the high-order QCD effects and concentrate on the moderate Q^2 behavior of the phenomena. In what follows we first outline the kinematics and two contrasting dynamical mechanisms. One is scattering from the "naive" ocean parton distribution, and the other is quark-pair production via "photon-gluon fusion." Thereafter we discuss implications for A-dependence studies and properties of the hadron final states. We also speculate on implications for hadroproduction processes.

II. Kinematics and Mechanisms

The two mechanisms to be discussed are shown in Fig. 1.

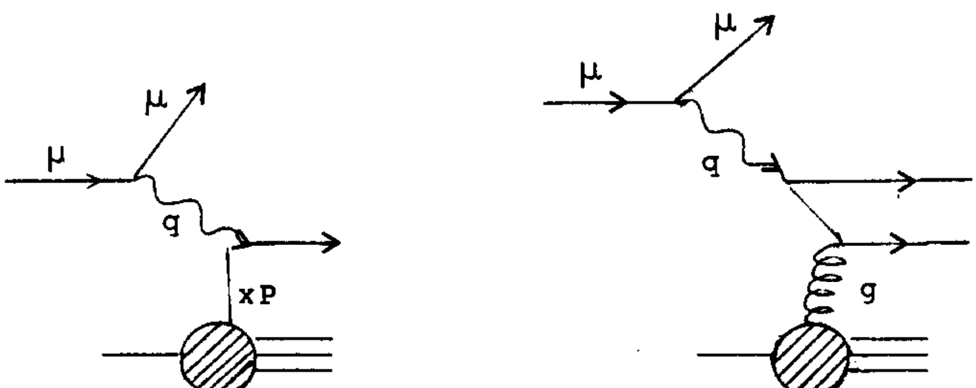

Fig. 1

Fig. 1 a) Naive parton model b) Photon-gluon fusion

The cross-sections for these may be easily computed and we record them here:

(a) Naive scattering from the "ocean" (u, d, s, ū, d̄, s̄)

$$\lim_{E_\mu \to \infty} \nu \frac{d\sigma}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^4} \left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} \right) \frac{F_q(x)}{3} \tag{2.1}$$

where $x^{-1}F_q(x)$ is the quark distribution summed over quark types $(u,d,s,\bar{u},\bar{d},\bar{s})$. We ignore charm because of the strong threshold dependence in the relevant range of Q^2. The scaling variable

$$x = \frac{Q^2}{2M\nu} \tag{2.2}$$

is interpreted as usual and is defined by the lepton kinematics. This comment is relevant for the photon-gluon fusion mechanism we now discuss.

(b) Photon-gluon Fusion

We obtain the cross-section from the analysis of charm electroproduction of Barger et al as quoted by Gollin, et al.[3] Setting $m_c = 0$ wherever possible, one obtains

$$\lim_{E_\mu \to \infty} \nu \frac{d\sigma}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^4} \cdot \frac{2}{9} \frac{\alpha_s}{\pi} F_g(x)\, \Phi(x) \tag{2.3}$$

with

$$\Phi(x) = \int_x^1 \frac{d\xi}{x}\left(\frac{x}{\xi}\right)^2 \frac{F_g(\xi)}{F_g(x)} \left[2\left(\ln \frac{m^2}{m_Q^2}\right)\left\{1-2\left(\frac{x}{\xi}\right)+2\left(\frac{x}{\xi}\right)^2\right\}-\left\{1-8\left(\frac{x}{\xi}\right)+8\left(\frac{x}{\xi}\right)^2\right\}\right] \tag{2.4}$$

Here $F_g(x)$ is the gluon distribution, summed over colors. We have taken this factor out in order to facilitate comparison of the normal "naive" ocean mechanism with the photon-gluon mechanism, since F_g and F_q (at small x) should not be too different, and since the only other differences in normalization are the factors α_s/π and $\Phi(x)$ appearing in the photon-gluon-fusion expression.

The heart of the matter is the factor Φ, which asymptotically contains logarithms galore. Hence $\frac{\alpha_s}{\pi}\Phi$ may be ≥ 1 and the two mechanisms competitive. The parameter

$$\xi = x\left(1 + \frac{m^2}{Q^2}\right) \tag{2.5}$$

appearing in the integral is the momentum fraction of the fused gluon; the parameter m is the mass of the $q\bar{q}$ system to which photon and gluon fuse.

The logarithm within the integral comes from the integration over the angular distribution of the produced $q\bar{q}$ pair at fixed mass m; more about that later. The remaining polynomials in x/ξ are inconsequential.

All this appears - and thus far is - quite straightforward. Nevertheless, when one contemplates A-dependence effects, subtleties arise, to which we now turn.

III. Space-Time Properties of the Amplitudes

Essential to both mechanisms is the fact that at small x these electroproduction amplitudes involve large longitudinal distances. This implies the relevance of a (generalized) "vector-dominance" mechanism. That is, we think of the evolution of the system in two stages:

i) Well upstream of the target nucleon or nucleus, the virtual photon dissociates into a $q\bar{q}$ pair, which then is free to possibly evolve further before arrival at the target.

It is easiest to use an old-fashioned perturbation-theory estimate of the energy difference ΔE between virtual photon and virtual $q\bar{q}$ system of mass m to estimate the propagation distance of this system.

$$\begin{aligned} \Delta t &\sim \frac{1}{\Delta E} = \frac{2\nu}{Q^2+m^2} = \frac{1}{xM} \cdot \frac{1}{\left(1+\frac{m^2}{Q^2}\right)} \\ &\approx \left(\frac{10^{-2}}{x}\right) \cdot \frac{1}{\left(1+\frac{m^2}{Q^2}\right)} \cdot (20\text{f.}) \end{aligned} \tag{3.1}$$

ii) Upon arrival at the target this virtual system interacts and is liberated, thus forming the final system of produced hadrons.

The origin of the distinction between production mechanisms lies in the structure of the virtual intermediate state, not at birth (there it is always a "bare" $q\bar{q}$ pair), but rather at arrival at the nucleon or nuclear target. Here we may distinguish three possible descriptions:

1) <u>"Naive" vector dominance</u>:

This option, which we shall rapidly dismiss, imagines the intermediate virtual system as a typical hadron, e.g. ρ, ρ', ρ'', ..., which is absorbed with a typical hadronic cross-section on the target. As pointed out by Gribov[4], such a model is inconsistent with scaling by a whole power of Q^2. Elementary calculations give, for absorption cross-section on a large target.

$$\sigma_T + \sigma_L = [1 - Z_3(Q^2)]\ \pi R^2 \tag{3.2}$$

Here R is the target radius, and $(1-Z_3)$ is the probability (which "runs" with Q^2) that the photon is a hadron at arrival. This is directly connected to the hadronic vacuum polarization contribution to the photon propagator; hence to the dimensionless colliding-beam cross-section parameter R:

$$1 - Z_3 = \frac{\alpha}{3\pi} \int_0^{\tilde{s}} \frac{dm^2 \; m^2 \; R(m^2)}{(Q^2 + m^2)^2} \tag{3.3}$$

For large Q^2, the factor $1-Z_3$ contains no intrinsic scale. Therefore we have, from dimensional analysis alone, the result that this "naive" vector-dominant picture predicts $\sigma_T + \sigma_L \geqslant \alpha R^2$ while scaling predicts $\sigma_T + \sigma_L \sim Q^{-2}$. Thus this picture is experimentally and conceptually wrong. We now turn to the mechanisms by which the models in question evade this result.

2) Photon-gluon Fusion

The photon-gluon fusion picture is perhaps the easiest to describe. It is kinematically similar to the QED Bethe-Heitler pair-production process, the only distinction being the virtuality of the incident photon. (This makes the typical mass of the produced pair order $\sqrt{Q^2}$, not $2m_e$.) For photon gluon fusion the role of the Weizacker-Williams Coulomb photon is of course replaced by the gluon cloud of the target.

A typical final state will leave the q and $\bar{q}$ (or e^+e) with comparable longitudinal momenta. Therefore the transverse momenta of the quarks will be $\lesssim m/2$ which in turn is typically of order $1/2 \sqrt{Q^2}$, i.e. large in the scaling limit. This in turn implies that, at arrival at the target, the transverse separation Δx_T of the q and $\bar{q}$ is small, or order $1/m$, or $1/\sqrt{Q^2}$. This has been verified by calculating, via "old-fashioned" light-cone perturbation theory, the wave function of the pair at arrival. It is also consistent with a simple classical geometrical estimate:

$$\Delta x_T \sim \left(\frac{p_t}{\nu}\right) x_L \sim \left(\frac{p_t}{\nu}\right)\frac{2\nu}{(Q^2+m^2)} \sim \frac{2m}{Q^2+m^2} \lesssim \frac{1}{Q} \tag{3.4}$$

This is in turn consistent with the quantum uncertainty relations.

Because, at arrival at the target, the q and $\bar{q}$ have hardly separated, they constitute a small color-dipole. Hence the interaction with the target is suppressed, relative to a typical strong interaction, by the square of the dipole moment. This gives a full power of Q^2, and restores the scaling behavior which was lost in the "naive" vector dominant approach. Furthermore, it should be a good first approximation to treat the intermediate $q\bar{q}$ system as free particles. Nonperturbative effects, e.g. formation of a string between q and $\bar{q}$, should be unimportant - although the <u>hard</u>-gluon radiative corrections of perturbative QCD, as usual, should be appended. At very large Q^2 and ν they do become important.

In analogy with QED and the Bethe-Heitler process, we expect the inelastic interaction of this color-dipole with nucleons in the nucleus to be incoherent and additive, due to the coulomb-like interaction of the dipole with the distinct gluon-clouds of the individual nucleons in the nucleus. Thus the basic dependence of this process on nuclear size is A^1, not $A^{2/3}$.

We also note that the typical final state consists, asymptotically, of two balanced high-p_T quark-jets and <u>no</u> beam-jet. This is hardly what is anticipated from ordinary parton-model considerations, where no final-state large-p_T secondaries are, to first approximation, expected.

3) <u>"Naive" Parton Model</u>

The parton-model description is usually not carried out in the laboratory frame but in, say, a center-of-mass frame of the system of target and incident lepton. In such a frame the momentum of exchanged photon is transferred to a single "ocean" quark. For a nuclear target this "ocean" quark is found in a cloud of longitudinal extent $\Delta x \lesssim 1f$, which is <u>large</u> compared to the thickness of the Lorentz-contracted pancake containing the nuclear matter. It is then expected that the number of ocean-partons per unit of transverse area (and per unit rapidity) saturates. Hence the electroproduction cross-section for lepton-nuclear scattering from wee ocean-partons would be expected to scale as $A^{2/3}$.

How does all this look in the laboratory frame? In that frame, the previous mechanism of photon-gluon fusion is in general inoperative, because the "naive" parton model includes only the nonperturbative strong interactions of partons of comparable momentum (or rapidity) and ignores long-range correlations in rapidity, such as the gluon-exchange interaction of the fast quarks in the color-dipole with the slow quarks in the target.

However, in the laboratory frame, the general vector-dominance picture still applies; the virtual photon first dissociates into the $q\bar{q}$ pair which then may evolve further.* What is different? It is simply that in this case the partition of virtual-photon longitudinal momentum to q and $\bar{q}$ is highly asymmetric - sufficiently asymmetric that the quarks no longer posses high p_T. When this is the case, on arrival the transverse separation Δx_T of q and $\bar{q}$ is no longer small, and can be of order $\langle p_T\rangle^{-1} \sim 1f$. Under these circumstances, there can be non-perturbative dynamical evolution during the propagation of the virtual state - e.g. string formation, creation of a cloud of wee partons, etc. We repeat, for $q\bar{q}$ initial configurations sufficiently asymmetric in longitudinal momentum, non-perturbative evolution can occur, and the system on arrival at the target may be "hadron-like" and be absorbed by the target with a typical nuclear mean free path.

The angular distribution of the virtual $q\bar{q}$ pair of given mass m, in its rest frame, is essentially isotropic. This means that the requirement of sufficient alignment along the beam-direction for the transverse momentum of q and $\bar{q}$ to be "typically" small, say $\lesssim \langle p_T\rangle \sim 300$ MeV, is simply that the center of mass angle θ^* satisfy

$$\theta^* \lesssim \frac{\langle p_T\rangle}{m} \tag{3.5}$$

or

$$1-\cos\theta^* \sim \frac{\langle p_T^2\rangle}{m^2} \tag{3.6}$$

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* It is quite appropriate that the initial ocean quark of the center-of-mass description is found, in the laboratory frame, in the negative-energy Dirac sea.

Isotropy implies a distribution uniform in cos θ^*. Hence the alignment probability is

$$\sim \frac{\langle p_T^2 \rangle}{m^2} \sim \frac{\langle p_T^2 \rangle}{Q^2} \tag{3.7}$$

It is this feature that restores the scaling behavior for the "naive" parton model[4].

The longitudinal momentum fraction z of the slow member of the quark pair is, under these conditions,

$$z = \frac{1-\cos\,\theta^*}{2} \lesssim \frac{\langle p_T^2 \rangle}{m^2} \tag{3.8}$$

We may again check via the classical calculation that the transverse separation Δx_T of the pair on arrival is large. It is

$$\Delta x_T \sim \frac{p_T}{p_L}\,(\Delta x_L) \sim \frac{\langle p_T \rangle}{z\nu} \cdot \frac{2}{Q^2+m^2} \sim \frac{2}{\langle p_T \rangle} \quad \frac{m^2}{(Q^2+m^2)} \tag{3.9}$$

which typically is large. This estimate again is compatible with the quantum uncertainty relations. With this amount of $q\bar{q}$ separation there is enough time available to dress the original $q\bar{q}$ system with wee partons - indeed with partons of momentum $\lesssim \frac{M}{x}$. (This is because the time required to dress the system with partons of momentum p is proportional to that momentum).

Thus the structure which arrives at the target is complex, and partially dressed. Its constituents which have momenta $\lesssim M/x$ are hadron-like. The remaining high-momentum portion is carried by the single leading quark which, <u>after</u> the collision with the target, may be expected to evolve and dress as does the final system in e^+e^- annihilation. Hence we recover the parton-model view of the structure of final state hadrons in longitudinal phase-space shown in Fig. 2.

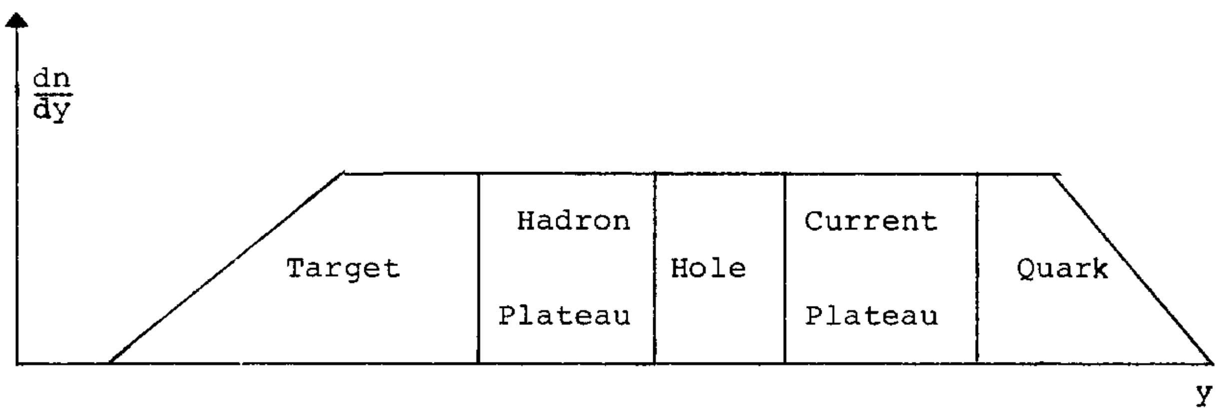

Fig. 2

We reiterate that the nuclear dependence of this process is expected to be $A^{2/3}$, and that the final state is predominantly low-p_T. This follows from the laboratory frame description as well as from the more conventional "infinite-momentum" frame of the parton model.

IV. Recapitulation of Experimental Consequences

A. Electroproduction

Let us now summarize the inferences we have made. First of all, at moderate Q^2 and wee $x \leq .02$, we expect the "naive" parton-model contributions to be the dominant contributor to electroproduction from a nucleon, with the photon-gluon contribution a relatively small correction (~ 10-20%). However, the A-dependence of the former is ~ $A^{2/3}$, while that of the latter is ~ A^1. Hence the two mechanisms may be quite comparable in very heavy nuclei.

While it may be difficult to untangle these contributions from measurement of structure functions alone, it should in principle be possible to do better by examination of the final state. In the idealized limit of small x and quite high Q^2 ($\gtrsim 100$ GeV^2) one should, for the naive parton-model mechanism, see only the typical beam-jet fragmentation, while for the photon-gluon mechanism one should see two leading, balanced high-p_T jets ($p_T \sim 1/2 \sqrt{Q^2}$ and no beam jet).

Unfortunately, until HERA is operating this idealization is unreachable. Nevertheless, there could still be some distinction in the final state properties for the two mechanisms even at lower Q^2. This is best examined via event simulations. But the mass of the $q\bar{q}$ system may have to be larger than 5-6 GeV in order to discern a jet orientation transverse to beam for photon-gluon fusion and parallel to beam for naive parton mechanism - just as it is in e^+e^- annihilation.

An additional distinction may occur in the multiplicity of low-momentum secondaries. The small color dipole present in the photon-gluon-fusion mechanism is not likely to suffer multiple collisions in traversing nuclear matter. Thus the final state may be relatively "diffractive," with less nuclear excitation and production of slow secondaries. Indeed, one may conjecture that the <u>only</u> produced hadrons in the typical photon-gluon fusion collisions - even in nuclei- are those associated with the fragmentation of the $q\bar{q}$ system. In other words the prescription is for produced $q\bar{q}$ system of mass m and cms production angle θ^* as follows: take the final hadron state for e^+e^- hadrons at cms energy m and jet angle θ^* and boost it until it has the momentum ν of the incident virtual photon. These are then conjectured to be <u>all</u> the particles produced in electroproduction via this mechanism.

In the "naive" parton mechanism, on the other hand, the $q\bar{q}$ system on arrival at a nuclear target does contain a low-momentum parton component and can be expected to suffer multiple nuclear collisions like an ordinary hadron. Thus the multiplicity of lower energy ($E \lesssim M/x$) hadrons should be characteristic of what is observed for a comparable π-nucleus collision.

B. Dijet Photoproduction

An upcoming Fermilab experiment[5] (E-683) which will use real photons is also very relevant to these considerations. According to vector-dominance phenomenology, roughly half the time a real photon which dissociates into hadrons essentially may be regarded, on arrival at a target, as a low-mass vector meson ρ, ω, ϕ. However there is a finite but small probability for the dissociated system to arrive as a massive $q\bar{q}$ system with symmetric momentum partition. If this occurs, the final system should again be of the photon-gluon-fusion character: two balanced high-p_T jets and no beam jet. The Fermilab experiment intends to observe this final state and test the QCD estimates of the production cross-section. The relevance of the remarks in this note is mainly in the final-state morphology. Will the production be a "diffractive" phenomenon with an A^1 dependence, as described above?

C. Heavy Quark Photoproduction and Electroproduction

Do these same considerations apply to the electroproduction of heavy quarks Q such as charm and bottom? The arguments clearly generalize as long as the mass of the produced $Q\bar{Q}$ system is large compared to the threshold value $2m_Q$; none of the previous kinematic estimates are significantly affected. When the mass is near threshold, one needs to estimate the size of the $Q\bar{Q}$ system on arrival as well as how much residual nuclear interaction occurs. Here it would seem that the theoretical issue can be largely finessed: the size can hardly be larger than that of typical $Q\bar{Q}$ onium systems. But it is known that ψ suffers very little absorption in nuclear matter; hence for low mass systems the nuclear dependence of photoproduction and/or electroproduction should be A^1. The only possibility for non-perturbative evolution seems to be for those $Q\bar{Q}$ configurations in which the relative p_T of Q and $\bar{Q}$ is small, $\lesssim$ 300 MeV. However these are probably power - law suppressed, just from phase-space arguments alone. We conclude that the process should have a linear dependence on atomic number, even in the "diffractive" limit.

D. Hadroproduction of Heavy Quarks

It is tempting to try to apply these ideas to heavy - quark hadroproduction, a subject which is today still somewhat confused. The two issues of relevance to the contents of this note are the dynamics of forward production and the question of A dependence. And, in brief, the question comes down to whether there appears anywhere a candidate for a nonperturbative production mechanism.[6]

After several false starts, I can offer only one candidate, illustrated in Fig. 3. As shown there an initial quark or gluon radiates a gluon which virtually converts to a $Q\bar{Q}$ heavy quark system which is in a color octet (Note this mechanism has no analogue in electroproduction). If the p_T of the $Q\bar{Q}$ system, and hence of the companion q or g is small, their relative impact parameter can become large, and a string and/or wee - parton cloud again will have the opportunity to form upstream of the target.

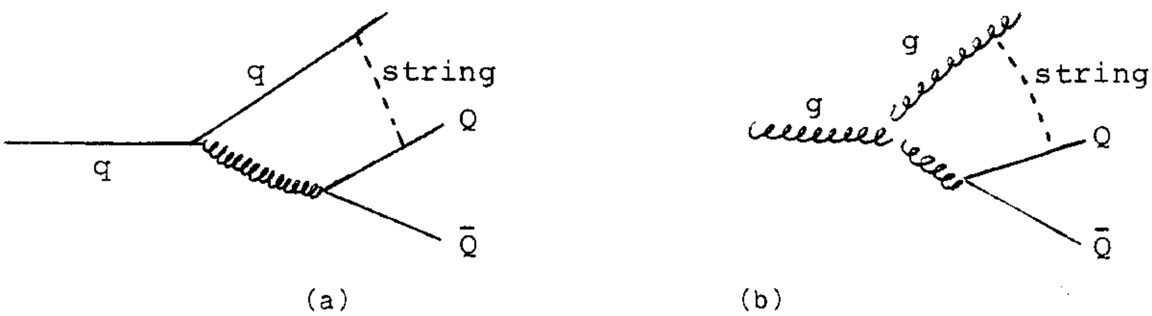

Fig. 3

This can lead to an $A^{2/3}$ contribution to the cross-section. Furthermore, since the momentum partition to the $Q\bar{Q}$ system is asymmetric, at least half the time the $Q\bar{Q}$ system will be leading, i.e. carry the full momentum of the initial q or g. Thus there may be an argument that production of a leading $Q\bar{Q}$ system is in part nonperturbative, and possesses an $A^{2/3}$ dependence. But the subject is tricky and I am not too sure of my ground here.

V. Conclusion

It is a real pleasure to dedicate these remarks to my colleague and friend, Jack Steinberger, whose work with neutrino production of hadrons has so much advanced our knowledge of the interior structure of the nucleon.

To be sure, the content of this paper is not very much directed to neutrino physics. There the situation at large x is in good shape. And, while there is still room for progress in the direction of higher energy and smaller x, it is probably the case that ultimately this subject is best attacked by muon and electron scattering experiments at the highest possible energies. I trust that this work will proceed with the same high standards and thoroughness as we have come to expect from Jack and his CDHS colleagues.

I thank Al Mueller for helpful comments and criticism.

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